Stability of a non-paraxial mirror and its application as a neutron source

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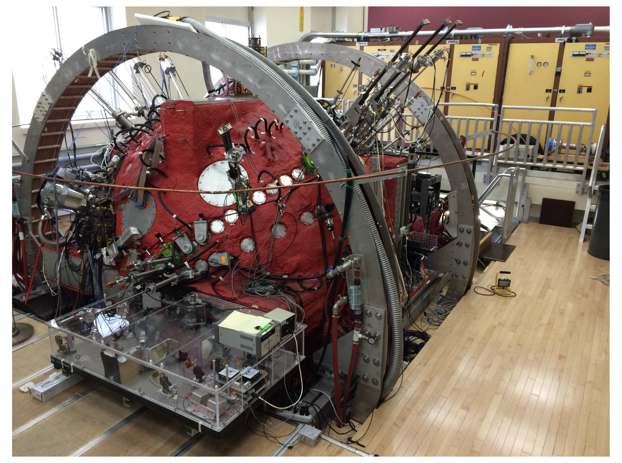
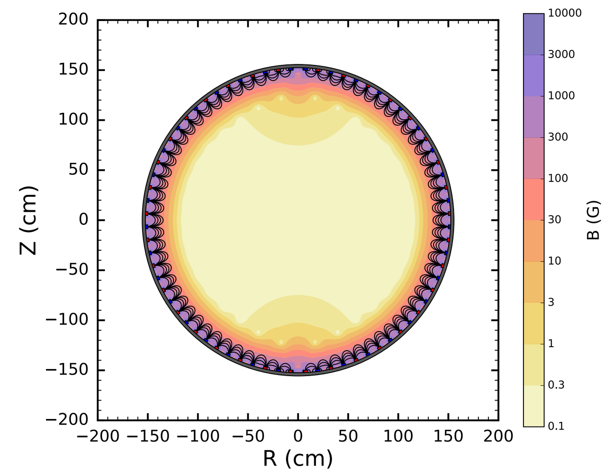
Background

1) Can we develop a fusion neutron source that takes advantage of new technologies, notably high temperature superconductors?

2) What happens if we make a mirror short and fat instead of long and skinny?

Big Red Ball (BRB) Vacuum Vessel





- 3 meter diameter spherical vacuum vessel
- Multi-dipole cusp confinement along the walls
- External Helmholtz coils can apply uniform magnetic field along Z

Axisymmetric Magnetic Field Solver

- No toroidal field
- Axisymmetric poloidal magnetic field $\vec{B} = \nabla\psi \times \nabla\phi = \frac{1}{R}\nabla\psi \times \hat{\phi} \quad \psi = RA_\phi$

A_ϕ given for the i[th] current loop in cylindrical coordinates by:

$$A_\phi(R,Z,R_i,Z_i) = \frac{\mu_0 R_i I_i}{4\pi((R+R_i)^2+(Z-Z_i)^2)^{1/2}} \frac{4}{k^2}[(2-k^2)K(k^2) - 2E(k^2)] \qquad k^2 = \frac{4RR_i}{(R+R_i)^2+(Z-Z_i)^2}$$

Linearity in I $\longrightarrow$ matrix multiplication:

$$\psi(R,Z) = \sum_i G_\psi(R,Z,R_i,Z_i)I_i \qquad B_R(R,Z) = \sum_i G_{B_R}(R,Z,R_i,Z_i)I_i \qquad B_Z(R,Z) = \sum_i G_{B_Z}(R,Z,R_i,Z_i)I_i$$

MHD Equilibrium Solver

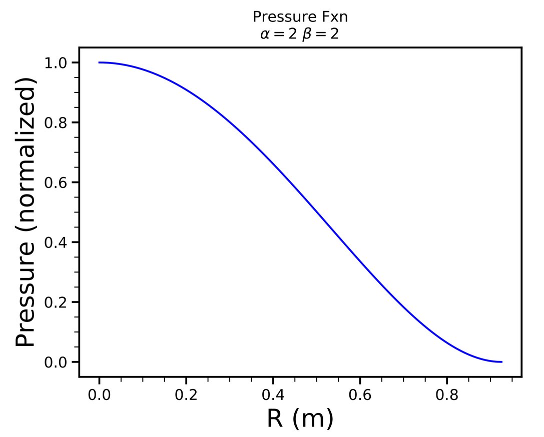
$$\vec{J}\times\vec{B} = \nabla P$$
$$(\vec{J}\times\vec{B})\times\vec{B} = \nabla P \times \vec{B}$$
$$\cancel{(\vec{J}\cdot\vec{B})\vec{B}}^{\,0} - B^2\vec{J} = \nabla P \times \vec{B}$$
$$\vec{J} = -\frac{\nabla P \times \vec{B}}{B^2}$$
$$\vec{J} = -\frac{\frac{dP}{d\psi}\nabla\psi \times (\frac{1}{R}\nabla\psi\times\hat{\phi})}{\frac{1}{R^2}|\nabla\psi|^2}$$
$$\vec{J} = -\frac{R\frac{dP}{d\psi}(\cancel{(\nabla\psi\cdot\hat{\phi})\nabla\psi}^{\,0} - |\nabla\psi|^2\hat{\phi})}{|\nabla\psi|^2}$$
$$\vec{J} = R\frac{dP}{d\psi}\hat{\phi}$$

Iterative Solver:

1) Compute magnetic field
2) Specify pressure profile
3) Map P to flux surfaces
4) Compute J from P'
5) Using J, compute ψ and add to ψ_{vac}
6) Repeat until convergence

- All assuming that we can write $P(\psi)$

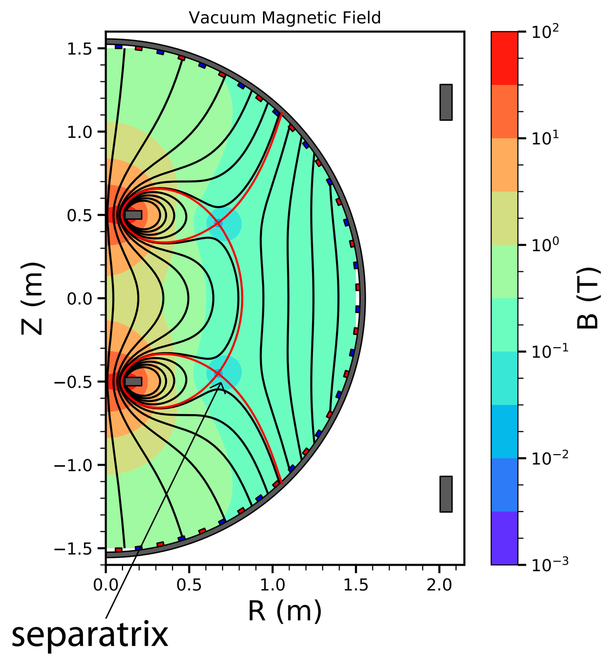
- Can iteratively update the pressure profile throughout

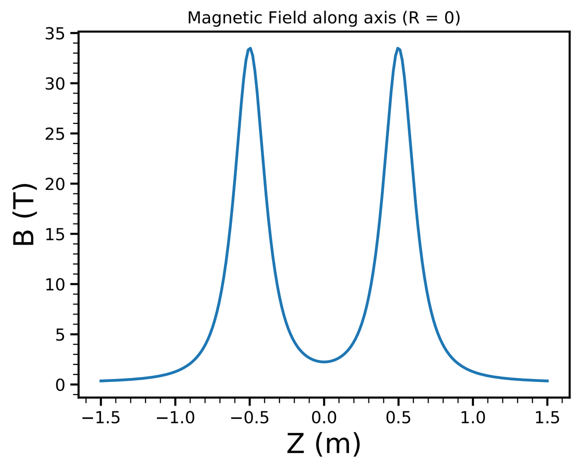


$$P = P_0\left(1-\left(\frac{r}{a_{sep}}\right)^\alpha\right)^\beta$$
$$a_{sep} = a_{separatrix}$$

Interchange Stability

Non-Paraxial Spherical Mirror





- Similar mirror "width" and "height"
- Lots of good magnetic field line curvature!
- **Easy to use HTS magnets!**

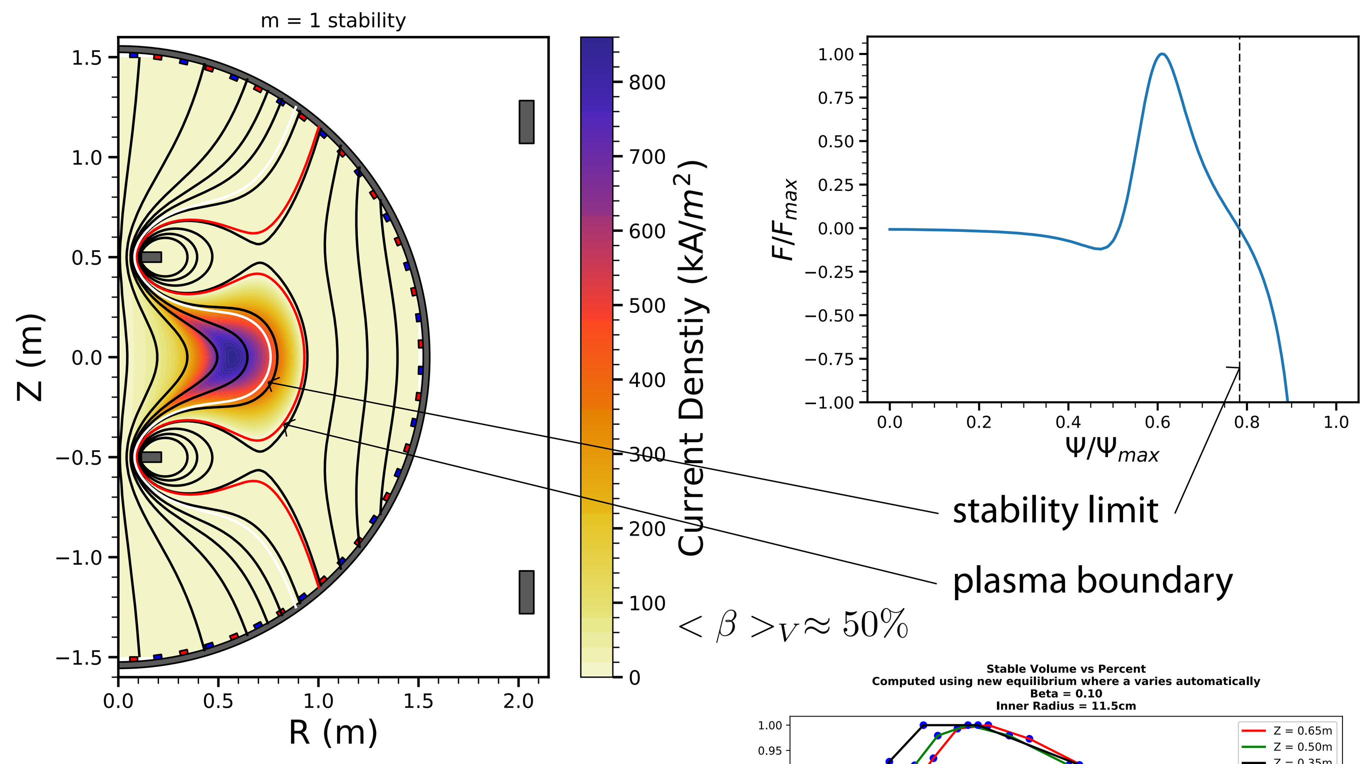
m=1 Flute Stability

$$U = \int \frac{dl}{B}$$

- Perturbation in potential energy for m=1, corresponding to a displacement of the plasma as a whole: $\delta W = A\int \Phi\left[\frac{dp}{d\Phi}\frac{dU}{d\Phi} + \gamma\frac{p}{U}\left(\frac{dU}{d\Phi}\right)^2\right]d\Phi$
- Any radial pressure profile can be composed from functions: $p = p_0\delta(\Phi - \Phi_0)$
- Substitute into the equation and integrating by parts yields the following:

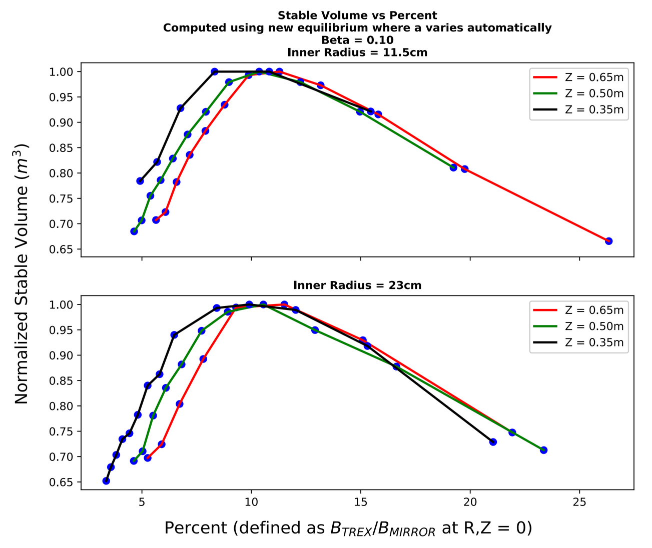
$$\delta W = Ap_0\left[\gamma\frac{\Phi}{U}\left(\frac{dU}{d\Phi}\right)^2 - \frac{d}{d\Phi}\left(\Phi\frac{dU}{d\Phi}\right)\right] \equiv Ap_0 F(\Phi)$$

- Stable if F > 0. Furthermore if a "stability region" exits at some intermediate values of magnetic flux, then any pressure distribution inside such a region is also stable.



How does geometry change stability?

- There exists an optimal magnetic field line geometry that maximizes the stable volume!



Ballooning Modes

- unperturbed plasma is assumed to be at rest
- unperturbed current density has only azimuthal component
- MHD stability analysis:
- perturbations proportional to $e^{(im\varphi - i\omega t)}$ $\quad m\rightarrow\infty$ (azimuthally localized)
- Force balance perpendicular and parallel to field lines. Linearizing the equations results in two second order differential equations:

$$B\frac{d}{ds}\frac{1}{\rho B}\frac{dX}{ds} + \omega^2\left(\frac{1}{\gamma p}+\frac{4\pi}{B^2}\right)X = -\frac{2\kappa\omega^2}{Br}Y$$
$$\frac{B}{4\pi}\frac{d}{ds}\frac{1}{Br^2}\frac{dY}{ds} + \left(\frac{\omega^2\rho}{r^2B^2}+\frac{2\kappa}{Br}\frac{\partial p}{\partial\Phi}\right)Y = -\frac{2\kappa}{Br}X$$

(s corresponds to distance along a field line)

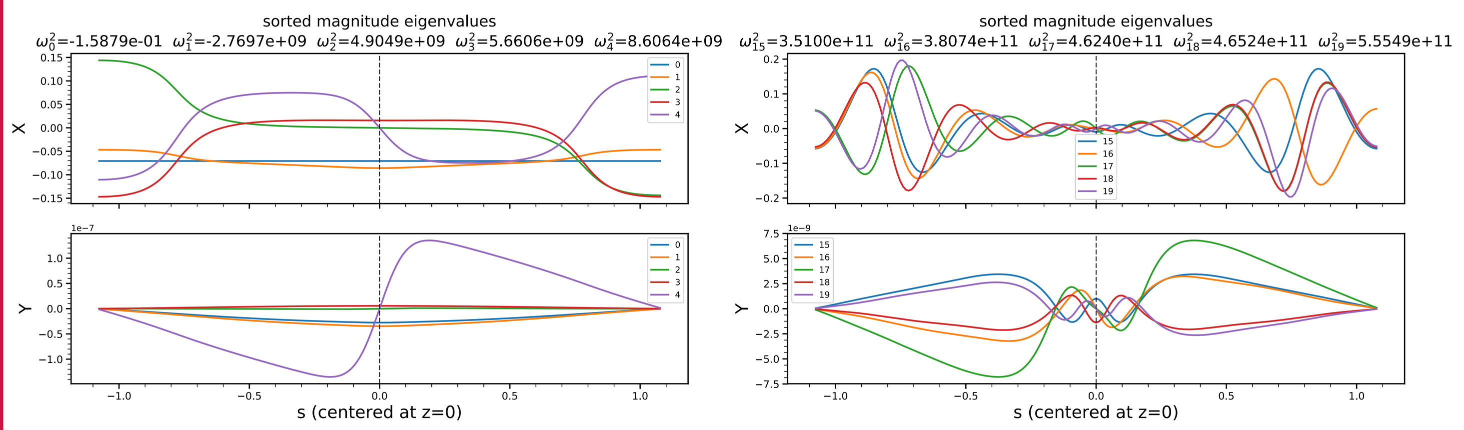
$X = \frac{\gamma p}{c}\nabla\cdot\delta\vec{v}$ "compressions along field line" $\qquad Y = r\delta E_\phi$ "displacement perpendicular to field line"

δ represents perturbations in the quantity

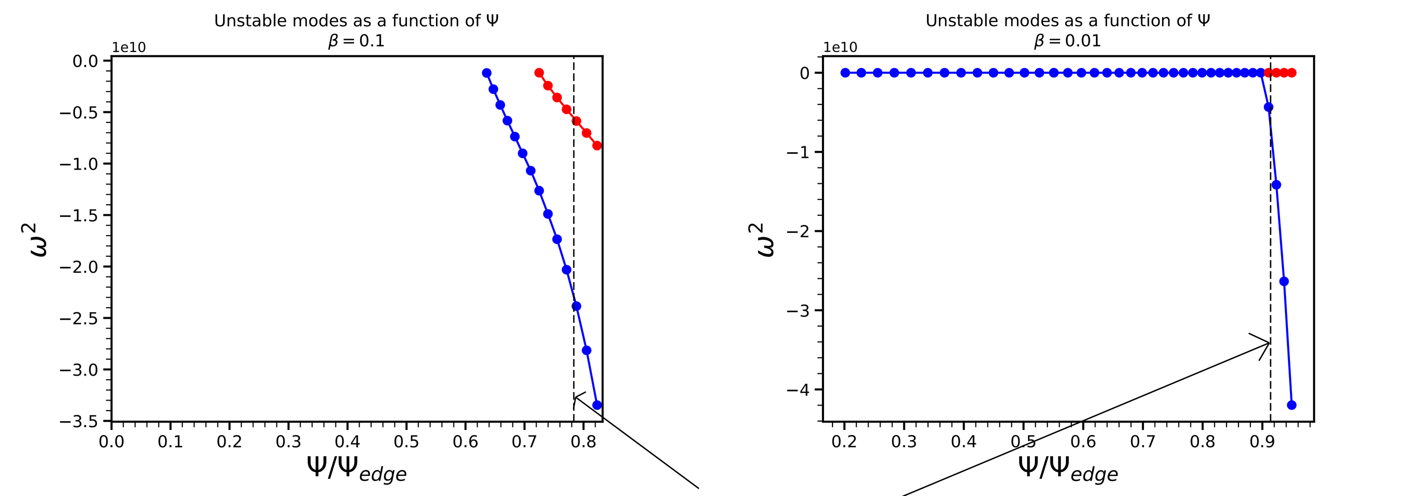
BCs: $\frac{dX}{ds} = 0$ (confined by rigid walls perp. to field lines); $Y = 0$ (ideally conducting wall)

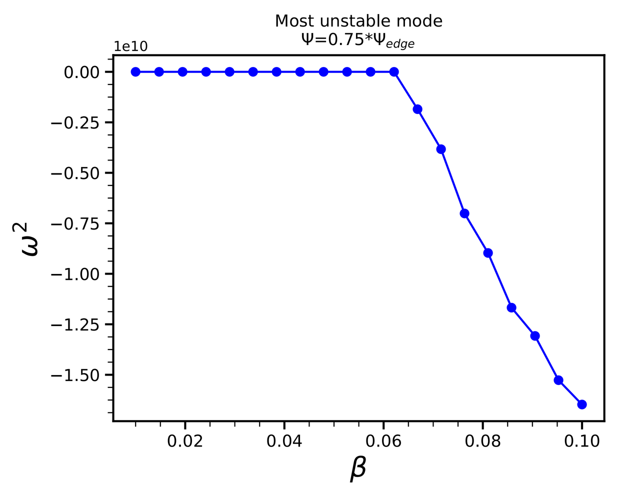
We have developed an eigenvalue solver that determines ω^2 eigenvalues and their respective eigenfunctions

An Example Solution $\frac{\psi}{\psi_{edge}} = 0.65$



How does the stability change?





m=1 stability limit

1) inner portion of the plasma is stable
2) unstable modes appear as you move out radially from the axis of symmetry
3) Stability is heavily dependent on the pressure profile
4) Stability limit decreases with increasing plasma beta

Fusion Neutron Source

Why create a neutron source?

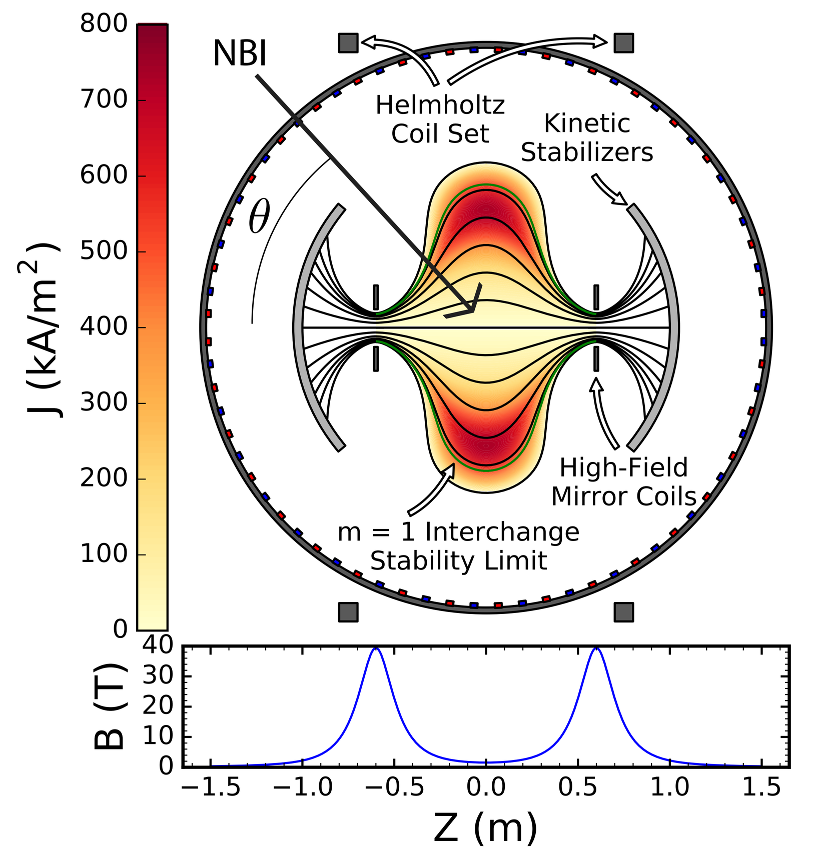
1) Intermediate milestone on the path to fusion energy
2) Neutrons are useful!
   - multi-billion-dollar need for high energy individual neutrons
   - medical isotope production
   - medical imaging
   - material testing and development

Can we use the spherical mirror for a fusion neutron source?

GENRAY (ray tracing)
- Provide equilibrium, n, T, B
- Choose wave type, frequency, launch point
- Returns wave path and power deposition in each species

CQL3D (bounce-averaged Fokker Planck code)
- Calculate energy deposition of waves from GENRAY
- NBI module for creating fast ions
- Simulates modifications to the distribution function due to external heating
- Calculates fusion reaction rates

Neutron Output



Equilibrium Parameters

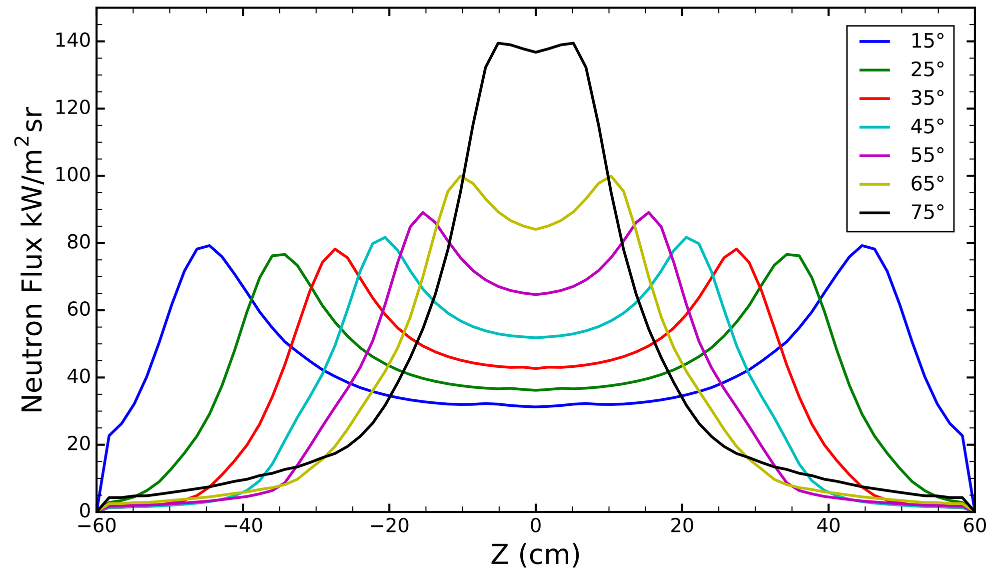
$n_e = 5\times10^{19}\ m^{-3}$

$T_i = 11$ keV D

$T_e = 1$ keV

10MW 75keV T NBI for various injection angles

All runs produce ~ 7×10^{16} neutrons/sec



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